THE GENERAL SOURCES OF NOISE IN VACUUM TUBES

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This paper discusses the genesis of noise in vacuum tubes. Simple if not rigorous derivations for various kinds of noise are given, and it is shown how the noise on an electron beam can be characterized. A few simple examples show how particular results can be obtained but nothing involving laborious mathematics is included. The effect of transverse velocities is discussed briefly, but no results are given.

An appendix gives some averages concerning the velocity distribution in electron beams.

1. Noise Figure

The noisiness of an amplifier is characterized by noise figure,[1,2] and in discussing noise in vacuum tubes it is well to start out with noise figure in mind.

Noise figure is defined in terms of Johnson noise,[3] that is, thermal noise. One way of characterizing Johnson noise is to say that a resistor or network at a temperature T is a noise source of available power P given by

$$P = kT\,B \tag{1.1}$$

Here k is Boltzmann's constant, T is absolute temperature and B is the bandwidth over which the noise power is measured. Numerically

$$P = 1.380 \times 10^{-23}\ T\ B \quad \begin{pmatrix}\text{watts, degrees}\\ \text{Kelvin, cycles per second}\end{pmatrix} \tag{1.1a}$$

P is called the available noise power because it is the maximum noise power which can flow into a passive load from the network. This maximum or available power will flow when the passive load impedance is the complex conjugate of the source impedance.

There are various consistent ways of defining the noise figure of an amplifier; we will do this as follows: Imagine that the input of the amplifier is connected to a source impedance held at a standard reference temperature T_r, usually taken as 20°C or 293°K. Part of the available power of the noise in the output of the amplifier will come from the Johnson noise in the source impedance; let us call this noise power P_t. If the power gain of the amplifier, defined as the ratio of the available power at the output to the available power at the source, is N, then

$$P_t = N k T_r B \tag{1.2}$$

There will in addition be a portion of output available noise power P_a which is not due to the Johnson noise from the source. The noise figure F is

$$F = \frac{P_t + P_a}{P_t} \tag{1.3}$$

$$F = 1 + \frac{P_a}{N k T_r B} \tag{1.4}$$

For narrow bandwidths, the amplifier noise P_a is usually proportional to the bandwidth B, so that over a narrow frequency range the noise figure is independent of bandwidth.

The noise figure of an amplifier does not depend on impedance of the load attached to the output but it does depend on the impedance of the source. The noise figure of a chain of amplifiers depends on the noise introduced by various component amplifiers. If the noise figure and available gain of the component amplifiers are known for the source impedances they see in the chain, the overall noise figure of the chain can be computed.

Let us now consider briefly some practical implications of the noise figure as defined by (1.3). Imagine, for instance, that the input of the receiver is connected to a lossless antenna system. The Johnson noise in the source (antenna) is that associated with the radiation resistance of the antenna. If the antenna is pointed at a perfectly absorbing (non-reflecting) body at temperature T, the noise temperature of the radiation resistance will be T. If the antenna is pointed toward the sky the radiation resistance temperature may be less than 10°K. If the antenna is pointed at a perfectly reflecting surface at a temperature T_R, so tilted that the antenna actually "sees" some body at temperature T, the noise temperature of the radiation resistance is T, not T_R. The situation can be summarized in a simple rule: If the antenna were used as a transmitting antenna, and a fraction p_1 of the power was ultimately absorbed after all reflections are taken into account by a body temperature T_1, a fraction p_2 by a body at temperature T_2, a fraction p_n by a body at temperature T_n, then the effective temperature T_a of the radiation resistance as a source of Johnson noise is

$$T_a = \Sigma p_n T_n \tag{1.5}$$

where the summation is taken over all bodies which the antenna sees.

Let us suppose that the antenna sees a background of effective temperature T_a and that it also sees a signal source of power P_s. What is the ratio of noise to signal, N/S? It is

$$N/S = \frac{N k T_a B + P_a}{N P_s}$$

$$N/S = \left(\frac{k T_r B}{P_s}\right)\left(\frac{N k T_r B - N k T_r B + N k T_a B + P_a}{N k T_r R B}\right)$$

$$N/S = \left(\frac{k T_r B}{P_s}\right)\left(F - (1 - T_a/T_r)\right) \qquad (1.6)$$

The first factor is the ratio of reference noise power to signal power. The second factor when expressed in db may be very different from the noise figure. Let us consider the extreme case of an antenna pointing at the sky, so that we can take T_a as zero, and so that the second factor becomes F-1. Below are given some corresponding values of F and F-1, expressed in db

F, db	(F-1), db
10	9.5
5	4.8
4	1.8
2	-2.4
1	-5.7
.5	-9.2

We see that in going down 3 db from a 4 db noise figure to 1 db noise figure the signal-to-noise ratio may be improved as much as 7.5 db. This is because in defining the noise figure we included the Johnson noise in a hypothetical source at a standard reference temperature T_r, and this hypothetical source is not in the actual system.

2. Johnson Noise

So far we have given an expression for Johnson noise but we have not discussed its physical significance. Consider any physical system which is in equilibrium, that is, all at the same temperature. Energy will continually flow back and forth from one part to another. Part of the energy may be mechanical (acoustic) energy; part may be radiant (electromagnetic) energy. Johnson noise is a name that engineers give to the electrical or electromagnetic part of this energy. For instance, suppose that two resistors of the same resistance R are interconnected by a lossless non-dispersive transmission line of length L and of characteristic impedance K equal to R, as shown in Fig. 1. Then a noise power kT B will flow along the line in each direction.

In fact, it was by considering this physical system that Nyquist was able to derive the expression for Johnson noise.[4] Classical statistical

mechanics tell us that in a system in equilibrium there is an energy kT associated with each mode of oscillation. Nyquist said, let us suddenly short the resistors at each end of the line, so that the line becomes a resonator with resonant modes having frequencies Δf apart, where Δf is given by

$$\Delta f = \frac{v}{2L} \tag{2.1}$$

Each of the modes will have an energy kT, half of which can be thought of as belonging to a wave flowing from left to right and half to a wave flowing from right to left. Hence the energy per unit length associated with the wave in one direction is

$$\frac{kT}{2L}$$

The power flow P in one direction associated with one mode is

$$P = \frac{kT}{2L} \; v$$

$$P = kT\Delta f \tag{2.2}$$

This is just the formula for Johnson noise, (1.1).

In deriving the expression for Johnson noise, thermal equilibrium was assumed. When is it valid to use the expression? As a practical matter, we can apply it in the case of any resistance or piece of passive lossy material. In the case of electromagnetic waves we can, through expression (1.5), arrive at an average or effective temperature for a radiation resistance. In the case of networks we can associate with each impedance an independent series noise voltage source V such that

$$V^2 = 4\ kTRB \tag{2.3}$$

where R is the resistive component of the impedance, or else an independent shunt current source I

$$I^2 = 4kTGB \tag{2.4}$$

where G is the conductive component of the impedance.

Expressions much like (2.3) and (2.4) occur in connection with the impedance and conductance of a space-charge-limited diode which has current flowing through it and is not a passive device. There is a reason for this, but it is wrong to apply the expression for Johnson noise to active, non-equilibrium devices.

3. Shot Noise

Shot noise is an important and elementary phenomenon in vaccum tubes. A flow of randomly emitted electrons from the cathode to the anode of a temperature-limited diode has a fluctuation called shot noise. This component I has a mean square value[5]

$$I^2 = 2e\, I_o\, B \tag{3.1}$$

Here e is the electron charge and I_o is the average current. Numerically

$$I^2 = 3.20\times10^{-19}\, I_o B \text{ (amperes, cycles per second)} \tag{3.1a}$$

In trying to see why (3.1) gives the shot noise for randomly emitted electrons it is simple (if not perfectly rigorous) to consider a periodic case in which we ask what the frequency components are of a current consisting, say, of one electron (of charge e) each T seconds. Such a series of electrons constitutes a regular spiked wave form as shown in Fig. 2. The spectrum of this current flow consists of a number of harmonics spaced Δf apart

$$\Delta f = \frac{1}{T} \tag{3.2}$$

The peak amplitude I_x of each of these harmonics is

$$I_x = 2e/T \tag{3.3}$$

Now let us consider that we have many different sets of electrons each second, each set producing its own spiked wave form as of Fig. 2, and each producing its own series of harmonics. Consider the case in which we already have a large number of electrons producing a noise component, of some chosen frequency, of peak amplitude I_m, and in which we add one more set of electrons, giving additional current I_x with a phase θ with respect to the existing noise I_m. Then, the square of the total current I is

$$I^2 = \tfrac{1}{2}\,(I_m + I_x\, e^{j\theta})\,(I_m + I_x\, e^{-j\theta})$$

$$= \tfrac{1}{2}\,(I_m^{\,2} + I_x^{\,2} + 2\, I_m I_x \cos\theta) \tag{3.4}$$

We see that by adding the randomly phased current I_x we always increase I^2 by an amount $I_x^{\,2}$, but if θ is chosen at random, the term $2\, I_m I_x \cos\theta$ is just as likely to be negative as positive, and, on the average, as we add more sets of electrons such terms do not contribute to the mean square noise current.

Thus, for p sets of electrons per second, the r-m-s value of I^2 each harmonic will be

$$I^2 = \frac{p}{2}(2e/T)^2$$

$$I_m^2 = \frac{2pe^2}{T^2} \tag{3.5}$$

The average current will be

$$I_o = \frac{pe}{T} \tag{3.6}$$

From (3.2), (3.5), and (3.6) we obtain

$$I^2 = 2e\, I_o\, \Delta F \tag{3.7}$$

This is just (3.1).

In a temperature-limited diode at moderate frequencies, for which transit-time is negligible, we can speak of shot noise in the diode (circuit) current. When transit-time is important we must distinguish between fluctuations in electron convection current or convection current density at a point and fluctuations in currents in circuits coupled to the electron stream.

In comparing (3.1) with expressions for other forms of noise it is of interest to write it in a little different form. Let M be the number of electrons passing per second. Then

$$I_o = M\, e \tag{3.8}$$

We can rewrite 3.1

$$I^2 = 2\, I_o^2\, \frac{B}{M} \tag{3.9}$$

In other words, the mean square shot noise current is twice the total electron current squared times the ratio of the frequency bandwidth to the frequency of passage of electrons.

4. Noise Different from Shot Noise

The noise in the electron convection current may be greater or less than shot noise; if it is different from shot noise, this implies some correlation between the times different electrons pass a given point.

For instance, suppose that electrons go by in pairs closely tied together, but that different pairs pass randomly. We then have in effect, not charges of charge e but charges of charge 2e, and the noise will be twice as great, that is

$$I^2 = 2(2e)\, I_o B \tag{4.1}$$

As another case, suppose that the electrons pass a point at equal intervals t, which for a current I_o would make the intervals

$$t = e/I_o = 1.60 \times 10^{-19}/I_o \tag{4.2}$$

Then, there will be no a-c component up to a frequency 1/t, which is so high as to be out of this world; at lower frequencies the current will be noiseless.

The noise current in a space-charge-limited diode operating at low frequencies is less than shot noise, and in connection with low-frequency diodes and triodes a noise reduction factor Γ^2 is used to relate the noise current to shot noise[6-9], so that the mean square noise convection current I^2 is given by

$$I^2 = \Gamma^2 \, 2e \, I_o B \tag{4.3}$$

Let us now consider what happens if we intercept part $(I_o - I_1)$ of the current I_o in an electron beam.[6,10,11] We will picture the noise in the beam as consisting of several parts: a pure shot noise associated directly with the remaining current I_1, of magnitude

$$\sqrt{2 \, e \, B \, I_1} \tag{4.4}$$

A compensating current distributed over the whole beam and serving to reduce this noise, of magnitude

$$\sqrt{2 \, e \, B \, I_1} \; (1 - \Gamma) \tag{4.5}$$

A pure shot noise directly associated with the intercepted current $I_o - I_1$, of magnitude

$$\sqrt{2 \, e \, B \, (I_o - I_1)} \tag{4.6}$$

A compensating current distributed over all the beam, of magnitude

$$\sqrt{2 \, e \, B \, (I_o - I_1)} \; (1-\Gamma) \tag{4.7}$$

Currents (4.4) and (4.5) are correlated and 180° out of phase; currents (4.6) and (4.7) are correlated and 180° out of phase, but currents (4.4) and (4.5) are uncorrelated with currents (4.6) and (4.7).

In the unintercepted part of the beam we will have current (4.4), I_1/I_o times current (4.5) 180° out of phase with (4.4), and (I_1/I_o) times (4.6) uncorrelated with the other currents. Thus, the total noise current I^2 in I_1 will be

$$I^2 = \left\{ \left(I_1^{1/2} - I_1^{1/2} (1 - \Gamma) (I_1/I_o) \right)^2 + (I_o - I_1) (1 - \Gamma)^2 (I_1/I_o)^2 \right\} 2e\, B$$

$$I^2 = \left(\frac{I_o - I_1}{I_o} + \frac{I_1}{I_o} \Gamma^2 \right) 2e\, I_1\, B \qquad (4.8)$$

Thus, noise in the intercepted current is less than pure shot noise by a factor

$$\left(\frac{I_o - I_1}{I_o} + \frac{I_1}{I_o} \Gamma^2 \right)$$

If the intercepted current $I_o - I_1$ is very small, the noise is very nearly the original beam noise plus shot noise in the intercepted current.

In this discussion it has been assumed that the compensating current for noise in a part of the beam is distributed over the whole beam. If the compensating current existed at exactly the same part of the cross-section as does the noise current there would be no partition noise. If the compensating current were spread out, but not over the whole beam, the partition noise might be less than given by (4.8).

5. Fluctuations in Velocity

The electrons leaving a hot cathode leave with various velocities, the overall velocity distribution being Maxwellian. This results in a fluctuation of the average velocity, which has been evaluated by various workers.[7,12,13]

Let a current I_n forming part of a current I_o of electrons leave with a velocity v_n. Then the average velocity u_o is defined as

$$u_o = \frac{1}{I_o} \sum I_n v_n \qquad (5.1)$$

Suppose we want to find the fluctuation in u_o due to fluctuations in the current I_n. By differentiating we obtain

$$d u_o = \left(- \frac{1}{I_o^2} \frac{dI_o}{dI_n} \sum_n I_n v_n + \frac{v_n}{I_o} \right) d I_n \qquad (5.2)$$

Now, if we change I_n by an amount $d I_n$, we also change I_o by an amount $d I_n$; hence

$$\frac{d I_o}{d I_n} = 1 \tag{5.3}$$

From (5.3), (5.2) and (5.1) we see that

$$d u_o = \frac{1}{I_o} (v_n - u_o) d I_n \tag{5.4}$$

Now, each velocity-class of electrons is emitted independently, so we may assume each fluctuation $d I_n$ to be an independent shot noise, given by (3.1). The corresponding mean square velocity fluctuation will be obtained by adding the squares of the d uo's, so that the mean square velocity fluctuation will be

$$v^2 = \Sigma (d u_o)_n^2$$

$$v^2 = \frac{2e}{I_o} \overline{\delta v^2} B \tag{5.5}$$

where $\overline{\delta v^2}$ is the mean square derivation in velocity, that is,

$$\overline{\delta v^2} = \frac{1}{I_o} \Sigma I_n (v_n - u_o)^2 \tag{5.6}$$

for a Maxwellian velocity distribution (see Appendix)

$$\overline{\delta v^2} = \frac{1}{2} (4 - \pi) \frac{kT_c}{m} \tag{5.7}$$

Here T_c is cathode temperature. Accordingly

$$v^2 = \frac{(4-\pi)e\ k\ T_c B}{m\ I_o} \tag{5.8}$$

We may wonder how I_o got into (5.5) and (5.8). Let us, following (3.8), use M to denote the number of electrons per second passing a plane. Then (5.5) can be rewritten

$$v^2 = 2 \overline{\delta v^2} \frac{B}{M} \tag{5.9}$$

we see that this is closely comparable to expression (3.9) for shot noise.

6. Velocity Distribution and the Potential Minimum

Consider the flow of electrons between the cathode and the plate of a parallel-plane diode in which the anode is positive with respect to

the cathode, as indicated in Fig. 3. Here c represents the cathode, m the potential minimum and a the anode; the curve indicates the voltage variation between anode and cathode.

Going away from the cathode toward the anode, the potential falls and the slower electrons are turned back. Finally a potential minimum is reached, beyond which the potential rises and no electrons are turned back. Let I_o be the current passing the potential minimum and I_c be the current emitted from the cathode,

$$I_o = I_c \quad e^{-\frac{e V_m}{kT_c}} \tag{6.1}$$

$$\frac{\partial I_o}{\partial V_m} = -\frac{e I_o}{kT_c} \tag{6.2}$$

There is also a relation between I_o and I_c and the distance from cathode to potential minimum and a relation between I_o, V_a and the distance between the potential minimum and the anode; these were given by Langmuir[14] and are to be found in standard books. If the diode is fed from a constant-current source so that I_o is fixed, V_m adjusts itself so that (6.1) is satisfied and V_a adjusts itself so that Langmuir's relations are satisfied.

Let us now consider low-frequency noise in this constant-current diode. If the potential minimum were held fixed in voltage with respect to the cathode, then the electrons would pass it quite independently and the current passing the potential minimum would be pure shot noise;

$$I^2 = 2 e I_o B \tag{6.3}$$

If the diode is to operate at constant current, V_m must fluctuate by an amount which will just cancel this current; from (6.2) we see that the mean square value of the amount by which V_m must fluctuate to cancel this shot-noise current is

$$V_m^2 = (2e I_o B) \quad \left(\frac{kT_c}{eI_o}\right)^2$$

$$V_m^2 = 2 kT_c \quad \left(\frac{kT_c}{eI_o}\right) B \tag{6.4}$$

The quantity (kT_c/eI_o) is the electronic resistance between the cathode and the potential minimum regarded as a diode. We should note the strong resemblance of (6.4) to the expression for Johnson noise

$$V^2 = 4 kT R B \tag{2.3}$$

Does not the fluctuation in V_m cause fluctuations in the average velocity of the electrons passing the potential minimum? Oddly enough, it does not. It is a very special property of the Maxwellian distribution of velocities that if the electrons are slowed down and some turned back, the remaining electrons have exactly the same average velocity as before; only the current is changed. Hence, the fluctuation in velocity at the potential minimum is not changed by the fluctuations in the voltage of the potential minimum and is, in fact, that given in Section 5

$$v^2 = \frac{(4-\pi)\, e\, k\, T_c B}{m\, I_o} \tag{5.8}$$

This fluctuation is uncorrelated with the fluctuation in V_m.

We have computed the fluctuation in the voltage between the cathode and the potential minimum. While the current entering the space between the potential minimum and the anode is constant, the velocity of the electrons fluctuates according to (5.8) and hence the total charge in this space fluctuates; because of this the voltage V_a will fluctuate. By using (5.8) in connection with the exact relations for electron flow in the space between the potential minimum and the anode, taking into account the fluctuation in the position of the potential minimum, we could presumably compute the noise fluctuation in the anode voltage V_a and hence the total noise fluctuation for the diode, which others have calculated exactly.

We can get an approximate answer by neglecting fluctuations in the position of the potential minimum and by neglecting the spread of velocities in the region between the potential minimum and the anode.[15] Llewellyn and Peterson have given relations covering this case,[16] and we find approximately (neglecting the average electron velocity at the potential minimum)

$$V_a^2 = \frac{3}{(e/m)}\ I_o\, R\, v^2 \tag{6.5}$$

Here R is the diode resistance computed according to Child's law

$$R = \frac{2}{3}\ \frac{V_a}{I_o} \tag{6.6}$$

By combining (5.8) and (6.5) we obtain

$$V_a^2 = 3\,(4-\pi)\, k\, T_c\, R\, B \tag{6.7}$$

$$V_a^2 = (.644)\,(4\, k\, T_c\, R\, B)$$

Again, we should compare (6.7) with the expression for Johnson noise, (2.3).

For widely-spaced diodes

$$R = \frac{2}{3}\,\frac{V_a}{I_o} \gg \frac{kT_c}{eI_o} \tag{6.8}$$

and hence

$$V_a^2 \gg V_m^2 \tag{6.9}$$

When this is so, the fluctuation current I^2 in a short-circuited diode is approximately

$$I^2 = (1.932)\,\left(\frac{kT_c}{eV_a}\right)\ (2e\,I_o B) \tag{6.10}$$

$$= \Gamma^2\, 2e\, I_o B$$

Hence, the space-charge noise reduction factor will be

$$\Gamma^2 = 1.932\,\frac{kT_c}{eV_a} \tag{6.11}$$

Typically, kT_c/e may be around 0.1, and hence, roughly

$$\Gamma^2 = \frac{0.2}{V_a} \tag{6.12}$$

Thus, at low frequencies and for a large anode voltage, Γ^2 is very small and the noise in the convection current of a diode is very much less than shot noise. The noise is so low because the potential minimum so fluctuates as to make the convection current passing it very nearly constant.

7. When Simple Relations Fail

At low frequencies, various writers have taken the velocity distribution of electrons into account in calculating the various properties, including noise properties, of tubes[6-9]. Further, we have seen in Section 6 how a correct multi-velocity relation for the potential minimum can be combined with relations based on electron flow without any spread in velocities to give approximately correct results.

What happens at frequencies so high that we can no longer neglect transmit time? For such frequencies we have a reasonably adequate theory

for electron streams with no velocity spread (single-valued-velocity theory) but almost no useful theory at all taking into account the actual spread in electron velocities.

Now, if electrons are accelerated by a voltage V_o, very nearly

$$\frac{\sqrt{\overline{\delta v^2}}}{u_o} = \frac{1}{2}\frac{kT}{eV_o} \tag{7.1}$$

Here $\overline{\delta v^2}$ is the mean square deviation in velocity and u_o is the mean velocity. Typically, for V_o = 1 volt, the ratio of the r.m.s. velocity spread to the mean velocity will be 1/20, and for higher voltages it will be even less. Hence, we have reason to believe that our single-valued velocity theory will be adequate for all portions of the electron beam which are at a potential higher than a volt or so. Experiment seems to bear this out.

Thus, it is only the region between the cathode and the potential minimum, and perhaps a little beyond it, for which we have no adequate theory.

In microwave triodes or diodes this region is extremely important and strongly influences the properties of the tubes. This shows up in several ways.

For instance, in 1941 the writer measured the rectifying properties of diodes at 10 cm wavelength and at 60 cycles. At 60 cycles the rectification in the sense of current change at constant d-c voltage was greatest near cutoff; at 10 cm wavelength the rectification rose with current, and it was very nearly proportional to transconductance. Now, Benham[17] has given a second-order single-valued-velocity treatment of the diode which predicts that rectification at very long transit angles should be just a constant times rectification as short transit angles; hence, in this case the actual multi-valued-velocity flow leads to behavior quite contrary to that predicted by single-valued-velocity theory.

Llewellyn has pointed out that at certain transit angles diodes should exhibit negative resistance; in the centimeter range, experimentally this effect is much weaker than single-valued-velocity theory would indicate, and sometimes it is absent. Again, multi-valued-velocity flow near the potential minimum is to blame.

Particularly, at very high frequencies the electrons which are turned back before reaching the potential minimum can pick up energy from the r-f field, and such electrons act both as a resistive load and as a source of noise.[19,20]

8. Where the Single-Valued-Velocity Theory Succeeds

In klystrons, traveling-wave tubes, and other beam devices, no r-f signal is applied near the potential minimum, and the velocity spread of electrons is negligible not only in the region of r-f interaction but in all of the electron beam to within a very short distance from the potential minimum.[21] Theory[13,22] and experiment both indicate that linearized or low-level single-valued-velocity theory is satisfactory in dealing with both noise and signal in this portion of the beam. However, to calculate noise in such tubes we need to know the noise current and the noise velocity at or near the potential minimum.

It is perhaps safe to take (5.8) as giving the noise velocity fluctuation at the potential minimum. What, however, are we to take as the noise current? We might assume pure shot noise, uncorrelated with the velocity fluctuation, but this leads to calculated noise figures which are higher than the best observed noise figures in traveling-wave tubes (about 5 db at 10 cm wavelength).

Watkins[23] proposes to estimate the noise figure by saying that the potential minimum fluctuates and reduces the noise convection current passing it just as it does at low frequencies, but not so completely, because of the bypassing effect at high frequencies of the capacitance between potential minimum and cathode. This leads to an expression for noise current at the potential minimum which is less the smaller $\lambda^2 J_o$ is, where λ is wavelength and J_o is the current density.

Peter has suggested that variations of space-charge-wave velocity across the electron beam near the potential minimum may nullify some of the noise reduction due to the potential minimum.

Actually, it is not clear just what the noise velocity and current should be near the potential minimum, nor whether or not they are correlated. However, we can deal with the beam beyond the potential minimum adequately by means of single-valued velocity theory, and it is comforting to know that the very simple assumption of full shot noise together with an uncorrelated velocity fluctuation characteristic of the Maxwellian velocity distribution over-estimates the noise somewhat (though not greatly) rather than underestimating it.

9. The Electron Beam as a Transmission System

Early calculations of noise in microwave tubes were based on a number of special solutions for a-c behavior of electron flow, and especially on the parallel-plane diode solutions of Llewellyn and Peterson and on the space-charge waves of Hahn and Ramo.[24-26] In using such an approach questions arose as to the possible effects on noise of varying the beam diameter, or of varying the diameter of a conducting tube surrounding the beam, or of accelerating the electrons with particular potential patterns.

It has been proposed more recently to treat the beam as a wave transmission system and to treat various regions in which the potential or the beam diameter changes as transducers which couple waves on portions of the beam. Various general relations and limitations can be arrived at by this means.[27-32]

The r-f excitation on an electron stream can be expressed in terms of a number of space-charge waves or modes of propagation. Actually, there are many such modes, but the evidence is that in narrow beams such as those used in low-noise tubes two modes or waves in which the a-c current and velocity are almost constant across the beam are most important, although other modes with more complicated patterns probably have some influence on noise figure.

We shall then consider noise in terms of these two modes only, disregarding the other modes. One of the modes is a slow mode, with a velocity a little less than that of the electrons; the other is a fast mode with a velocity a little greater than that of the electrons.

Very nearly, the phase constants β_1 and β_2 of the two modes are given by

$$\beta_1 = \frac{\omega}{u_o} + \frac{\omega_q}{u_o} \tag{9.1}$$

$$\beta_2 = \frac{\omega}{u_o} - \frac{\omega_q}{u_o} \tag{9.2}$$

Here ω_q is the effective plasma frequency, which is somewhat less than the plasma frequency. In connection with these modes, we can define a voltage-like quantity V

$$V = - \frac{v\, u_o}{e/m} \tag{9.3}$$

Here v is a-c velocity and e/m is the charge-to-mass ratio of the electron, a positive quantity. If I is the electron convection current, the power flow in the beam, P, is

$$P = \frac{1}{2} \quad (VI^* + V^* I) \tag{9.4}$$

Each mode has a characteristic impedance; the characteristic impedances for the two modes, K_1 and K_2, are

$$K_1 = \quad - 2 \frac{\omega_q}{\omega} \quad \frac{V_o}{I_o} \quad = - K_o \tag{9.5}$$

$$K_2 = \quad 2 \frac{\omega_q}{\omega} \quad \frac{V_o}{I_o} \quad = K_o \tag{9.6}$$

Here V_o and I_o are d-c beam voltage and current. These relations define K_o.

We see that the characteristic impedance for the slow wave is negative. This means that the slow wave has negative power flow; that is, the electron stream has less energy in the presence of the wave than in the absence of the wave. This comes about because the electrons are bunched together in regions of less than average velocity. The negative power flow of the slow space-charge wave was pointed out by L. J. Chu.[38]

Because the two waves have opposite energy power flows, when both have equal amplitudes the net power flow is zero, and we have a standing-wave pattern on the electron stream analogous to the standing wave pattern on a transmission line.

10. Noise Patterns on an Electron Beam

Let us disregard all but two modes of propagation on an electron beam; the pair consisting of a slow, negative-power wave and a fast, positive-power wave. We have described the source of noise in the electron stream as current and velocity fluctuations near the cathode. Let us ask first, what sort of excitation on an electron beam would a current fluctuation (I) alone, or a velocity fluctuation (V) alone, set up?

There is no power flow associated with I or V alone; hence, current fluctuation alone or velocity fluctuation alone will set up a standing-wave pattern on the beam; if we plot the envelope of the current of such a pattern vs distance it will look as shown by the solid or dashed lines of Fig. 4. The zeroes of each pattern are a distance L apart given by

$$L = \frac{\pi u_o}{\omega_q} \tag{10.1}$$

Let us now consider the overall pattern produced if we assume two uncorrelated noise sources, so that the two together result in no power flow. These noise sources will then set up two uncorrelated standing-wave patterns. The envelopes of the currents of these standing-wave patterns will be as shown by the solid and dashed lines of Fig. 4. The noise currents at the maxima of the patterns are I_1 and I_2.

Suppose, now that the beam passes from a region a through some lossless region, such as a series of velocity jumps, into a region b. If we know the noise standing-wave pattern in region a, what can we say of the noise standing-wave pattern in the region b?

The electron beam is a linear transmission system. For currents of a narrow frequency range, a current component I at one point causes another current I' further along the beam, and I' is a constant times I at some slightly earlier time. The constant and the delay will be the same whether

I is a sinusoidal current or a narrow-band noise current. If two currents are present simultaneously, the total current that they cause is the sum of the currents they would cause if they were present separately; that is, it is a sum of constants times component currents.

The fact that the relations between currents at two points in a beam must hold for sinusoidal currents of the same frequency as well as for noise currents enables us to establish certain relationships which must hold if energy is to be conserved. To see how this is so, let the maximum r.m.s. currents of the two standing-wave patterns I_1 and I_2, be sinusoidal currents of the same frequency.

For each standing-wave pattern individually the current is 90° out of phase with the voltage, so the product of the current and the voltage of one standing-wave pattern gives no power flow. However, the current of one pattern times the voltage of the other pattern can give power flow for excitations of the same frequency. Referring to Fig. 4 we easily see that the magnitude of this power flow must be

$$K_o \, |I_1||I_2|| \sin \theta \, || \sin \phi_\ell \, | \tag{10.2}$$

$$\phi_\ell = \frac{\omega_q \ell}{u_o} \tag{10.3}$$

Here $|I_1|$ is interpreted as the r.m.s. magnitude I_1. θ is the phase angle between I_1 and I_2. The voltage due to I_2 will be in phase with I_1 if the currents are 90° out of phase. The maximum of the voltage standing wave pattern associated with I_2 will be at the peak of the current pattern of I_1 if $\phi_\ell = \pi$.

If power is to be conserved we must have for the two regions a and b

$$K_{oa}|I_{1a}||I_{2a}||\sin \theta_a||\sin \phi_{\ell a}|$$

$$= K_{ob}|I_{1b}|I_{2b}||\sin \theta_b||\sin \phi_{\ell b}| \tag{10.4}$$

Now neither ℓ_b nor $|I_{2b}|$ will change just by changing the phase angle of the current I_{2a}; hence, if (10.4) is to hold we must have

$$|\sin \theta_a \, | = | \sin \theta_b \, |$$

and this leaves us with the relation

$$K_{oa}|I_{1a}||I_{2a}|| \sin \phi_{\ell a} |$$

$$= K_{ob}|I_{1b}||I_{2b}|| \sin \phi_{\ell b} | \tag{10.5}$$

Relation (10.5) must hold also when I_{1a} and I_{2a} are uncorrelated noise currents giving rise to uncorrelated noise current I_{2a} and I_{2b}. When I_1 and I_2 are uncorrelated, the square of the total current at any point along the pattern is the sum of the squares of the two uncorrelated current components. Thus, measuring z from the maximum of I_1, we have

$$I^2 = |I_1|^2 \cos^2 \phi + |I_2|^2 \cos^2 (\phi + \phi_\ell) \tag{10.6}$$

where

$$\phi = \frac{\omega_q}{u_o} z \tag{10.7}$$

From (10.6) we have

$$I^2 = \frac{1}{2} \left[|I_1|^2(1 + \cos 2\phi) + |I_2|^2(1 + \cos 2(\phi + \phi_\ell)) \right]$$

$$I^2 = \frac{1}{2} \left[|I_1|^2 + |I_2|^2 + (|I_1|^2 + |I_2|^2 \cos 2\phi_\ell) \cos 2\phi \right.$$

$$\left. + |I_2|^2 \sin 2\phi \right] \tag{10.8}$$

We see that the quantity in the brackets has a constant part

$$|I_1|^2 + |I_2|^2 \tag{10.9}$$

and a variable part of amplitude

$$(|I_1|^4 + |I_2|^4 + 2|I_1|^2 |I_2|^2 \cos 2\phi_\ell)^{1/2} \tag{10.10}$$

Hence, the pattern has a maximum value $|I_{max}|^2$ and a minimum value $|I_{min}|^2$ given by

$$2\ |I_{max}|^2 = |I_1|^2 + |I_2|^2 + (|I_1|^4 + |I_2|^4 + 2|I_1|^2|I_2|^2 \cos 2\phi_\ell)^{1/2} \qquad (10.11)$$

$$2\ |I_{min}|^2 = |I_1|^2 + |I_2|^2 - (|I_1|^4 + |I_2|^4 + 2|I_1|^2\ |I_2|^2 \cos 2\phi_\ell)^{1/2} \qquad (10.12)$$

We observe that

$$4\ |I_{max}|^2|I_{min}|^2 = 2\ |I_1|^2|I_2|^2\ (1-\cos 2\phi_\ell) \qquad (10.13)$$

$$|I_{max}|^2|I_{min}|^2 = \ |I_1|^2|I_2|^2\ \sin^2 \phi_\ell$$

By comparing (10.13) with (10.5) we see

$$K_{oa}|I_{max}|_a|I_{min}|_a = K_{ob}|I_{max}|_b|I_{min}|_b \qquad (10.14)$$

Thus, if an electron beam comes from a particular gun which has two uncorrelated noise sources, it has on it a noise standing wave with maximum and minimum currents I_{max} and I_{min} and

$$K_o\ |I_{max}|\ |I_{min}| = \text{constant} \qquad (10.15)$$

Here the constant does not depend on any lossless regions through which the beam passes.

Further, the noise current at the maximum is uncorrelated with the noise current at the minimum. To see this, we break the standing-wave pattern up into two components., $I_{max} \sin \phi$ and $I_{min} \cos \phi$. Here ϕ is measured with respect to the position of the minimum. I_{max} and I_{min} cannot have an out-of-phase correlation, for then there would be power flow. The mean square current $\overline{I^2}$, that is, the time average of I^2, will be

$$\overline{I^2} = \overline{(I_{max} \sin \phi + I_{min} \cos \phi)^2}$$

$$= \overline{I_{max}^2}\ \sin^2 \phi + \overline{I_{min}^2} \cos^2 \phi$$

$$+2\ \overline{I_{max}\ I_{min}}\ \sin \phi \cos \phi$$

$$\overline{I^2} = \frac{1}{2}\left[(\overline{I_{max}^2} + \overline{I_{min}^2}) - (\overline{I_{max}^2} - \overline{I_{min}^2})\cos 2\phi + \overline{I_{max}\ I_{min}}\ \sin 2\phi\right] \qquad (10.16)$$

At $\phi = 0$, where we have assumed the minimum to occur, $\cos 2\phi$ is stationary but $\sin 2\phi$ is not. Thus, unless

$$\overline{I_{max}\, I_{min}} = 0 \tag{10.17}$$

the minimum cannot occur at the minimum; and hence (10.17) must hold; that is, I_{max} must be uncorrelated with I_{min}.

Thus, the noise standing wave pattern due to two uncorrelated, zero-power noise sources consists of a standing wave pattern having a current I_{max} at the maximum and I_{min} at the minimum; I_{max} is uncorrelated with I_{min}, and the product of these times the beam impedance K_o is the same regardless of what lossless regions the beam may have passed through.

We note that at the maximum I_{max} has zero voltage associated with it and I_{min} has a voltage of magnitude V_{min} associated with it,

$$V_{min} = \frac{1}{K_o} I_{min} \tag{10.18}$$

V_{min} is greatest where I_{max} is greatest. Hence, relation (10.14) says that

$$|I_{max}||V_{min}| = |I_{min}||V_{max}| = \text{const.} \tag{10.19}$$

We further note that I_{max} and V_{min} are uncorrelated, as are I_{min} and V_{max}.

Suppose we assume that the noise sources are shot noise and a velocity fluctuation according to (5.8) at the potential minimum. Then (10.19) must hold at the potential minimum as well as anywhere else in the beam, and, noting the definition (9.3) of V, we have

$$|I_{max}||V_{min}| = \left[(2\, eI_o B)\, \frac{u_o^2}{(e/m)}\, \frac{(4-\Gamma) ek\, T_c B}{m\, I_o} \right]^{1/2} \tag{10.20}$$

We are in some trouble because our ideas of space-charge waves apply accurately only to beams without any spread in velocity. How are we to interpret u_o? If we interpret as the mean square velocity of electrons passing the potential minimum,

$$u_o^2 = \frac{2k\, T_c}{m} \tag{10.21}$$

From (10.21) and (10.19) we get

$$|I_{max}||V_{min}| = 2\sqrt{(4-\Gamma)}\; k\, T_c\, B \tag{10.22}$$

$$|I_{max}||V_{min}| = 1.84\ k\ T_c\ B \tag{10.22a}$$

We have erred in treating a multi-velocity flow in the region of the potential minimum as a single-velocity flow. Further, we have disregarded any noise reduction of convection current by action of the potential minimum. To take these effects into account we may write

$$|I_{max}||V_{min}| = 2\ \alpha\ k\ T_c\ B \tag{10.23}$$

Here α is around unity for pure shot noise at the potential minimum and is less in the case of noise reduction by the potential minimum. However, additional independent noise sources, such as partition noise, can result in an α greater than unity.

By use of space-charge-wave amplification and deamplification we can make I_{max} large or small.[33] If we make

$$V_{min} = K_o\ I_{max} \tag{10.24}$$

the current maximum and the current minimum will be equal and

$$|I_{max}|^2 = |I_{min}|^2 = \frac{2\ \alpha\ k\ T_c B}{K_o} \tag{10.25}$$

In this case, we have at the "maximum" or the "minimum" two uncorrelated slow-wave components, each of amplitude $|I_{max}|^2$ and two uncorrelated fast wave components each of amplitude $|I_{max}|^2$. The total power P_s slow wave component is

$$P_s = -\alpha\ k\ T_c B \tag{10.26}$$

For other cases the noise power in the slow wave will be greater.

In all of this discussion, it has been assumed that the net noise power in the beam is zero. Haus has considered the case in which the noise power flow is not zero.[35]

11. Interaction of Space-Charge Waves with Circuit Waves

In a traveling-wave tube the space-charge waves of the electron stream are coupled to the forward wave on the electromagnetic circuit. In low-C tubes we can neglect coupling between the space-charge waves and the backward circuit wave.

The result of the coupling between the space-charge waves and the forward circuit wave is that the three waves combine to form three new waves.

These may all be unattenuated, but when the velocity of the circuit wave is near to that of the slow space-charge we get an unattenuated wave, an increasing wave, and a decreasing wave. The unattenuated wave has positive power flow; the increasing and the decreasing waves individually have no power flow, but when they are present together there is a power flow proportional to the product of their amplitudes times the sine of their relative phase. The phase velocities of the two waves are the same and as one increases exponentially as fast as the other decreases exponentially, this product is constant with distance.

Without going through elaborate calculations, we can easily see how the things we have learned about noise can be used to obtain some results concerning the noise performance of traveling-wave tubes.

11.1 Optimum Noise Figure of a Lossless Traveling Wave Tube With Large Space Charge

When the space-charge parameter QC of a traveling wave tube is large the space-charge waves are well separated in velocity, and over the range of electron velocities for which there is gain, we can neglect coupling between the fast space-charge wave and the circuit wave. In this case, the slow space-charge wave and the forward circuit wave combine to give the increasing wave and the decreasing wave; in each wave the positive power flow of the circuit component is equal to the negative power flow of the space charge component.

If at the input the circuit is matched to a source of temperature T_r, the noise input power P_c in the input circuit wave is, from (1.1)

$$P_c = k T_r B \tag{11.1}$$

The minimum possible noise input power in the slow wave is given by (10.26). The square of the amplitude of the increasing wave excited by a given power in either of the waves which combine to form it is proportional to the absolute value of the power in that wave. Hence, if we consider the amplitude of the increasing wave as giving the output of a high-gain traveling-wave tube the noise figure F will be given by

$$F = \frac{P_c}{P_c} + \frac{P_s}{P_c}$$

$$F = 1 + \alpha \frac{T_c}{T_r} \tag{11.2}$$

This is the optimum noise figure for a lossless traveling-wave tube with large QC.

11.2 Two Theorems About Noise Cancellation

It would be highly desirable to build a traveling-wave tube such that the electromagnetic input would excite an increasing wave, but the noise in the electron stream would excite only some combination of the decreasing and the unattenuated waves. If we succeeded in this, the noise introduced by the tube could be made as small relative to the signal as desired, merely by making the tube long enough. Can we accomplish this by means of some special structure near the input end of the tube?

We can represent the noise on the electron stream at some reference point by means of a velocity fluctuation described by V and a current fluctuation I; we have seen that neither can be zero. Because the system is linear, superposition applies, and the amplitudes of the growing, attenuated and unattenuated waves which are excited are the sums of the amplitudes excited by I and V independently.

Suppose that V = 0. Then the beam carries no power. Thus I cannot excite the unattenuated wave, for that wave carries power. Let us assume that it excites the decreasing wave alone, which, when present alone, carries no power. So far there is no contradiction, and we can believe that it is possible to arrange matters so that the current alone does not excite the increasing wave.

Suppose we have so arranged matters that I does excite the decreasing wave only. Consider what happens when I = 0. Can V alone excite the decreasing wave only if I alone excites the decreasing wave only? If it can, then V and I together must excite the decreasing wave only. But suppose V and I are of the same frequency and in phase. Then the beam carries power, But, the decreasing wave alone cannot carry power, and hence what we have assumed is impossible. If the I excites the decreasing wave only, then V must excite at least a component of the growing wave.

Hence, we have our first theorem about noise cancellation.

Theorem I: If the current fluctuation I at a given point in the beam does not excite the increasing wave then a fluctuation in V will. This theorem is true if V and I are interchanged in its statement.

Let us now assume that I excites only the decreasing wave and that the circuit input excites, among other things, the increasing wave. Suppose that V is zero and that the beam convection current I and the circuit wave feeding the tube are of the same frequency. Suppose we vary their relative phases. This does not change the power of the beam entering the tube (which is zero if V = 0) nor the power of the circuit wave entering the tube. However, the power represented by the traveling-wave tube waves has a term which is the product of the amplitudes of the increasing and decreasing waves times the sine of the phase angle between them, and this phase angle will vary, and the power will vary, as we vary the phase between the beam current I

and circuit excitation. Hence, our assumption that I excites the decreasing wave only and that the circuit input excites the increasing wave leads to a contradiction, and we have a second theorem.

Theorem II: If the noise convection current I excites the decreasing wave only, the circuit input cannot excite the increasing wave.

This theorem is true if we replace I by V.

12. Transverse Effects

A narrow electron beam can interact with a slow circuit wave which has zero longitudinal field along a plane or cylindrical surface through the beam and produce gain. If the beam had zero width and no transverse velocity on entering the circuit, it could induce no noise in the circuit.

It is possible to analyze in detail the noise-figure of transverse-field tubes.[36] The noise inputs consist of fluctuations of beam position and of two components of velocity which can be evaluated by a direct adaptation of the methods of Section 5. It is found that the noise figure is not better than that for a longitudinal-field tube with no noise reduction by the potential minimum unless an aperturing system is used in the transverse-field tube which eliminates some electrons and thus reduces the width of the beam and the angular spread of electrons in the beam.

It seems likely that the transverse thermal motions of electrons add to the noise of conventional traveling-wave tubes in more than one way. The transverse velocities can excite noise fluctuations on the electron stream. The transverse velocities can also modify the nature of the waves on the electron stream. Hitherto all calculations of waves on electron streams have been based on laminar flow rather than on a more realistic sort of flow including transverse velocities of electrons.[39]

13. Final Comments

Looking back, we see that we started with the noise figure as a criterion of goodness for vacuum tubes. The noise figure was defined in terms of Johnson noise power, and it was related to signal-to-noise ratio in a microwave system.

Next, four sorts of noise of importance in vacuum tubes were considered and expressions for them were derived by simple if not entirely rigorous means; these sorts of noise are Johnson noise (thermal noise), shot noise, partition noise and velocity fluctuation.

The effect of shot noise and velocity fluctuation was illustrated in a simple case, that of the low-frequency diode. It was then pointed out how such a simple treatment fails at high frequencies because we have no adequate treatment of the potential minimum region for high frequencies.

We do, however, have a single-valued-velocity treatment of the electron stream which is valid for all portions of the beam at which the potential is more than a volt or so positive with respect to the cathode.

While even the single-valued velocity treatment of electron flow can be very complicated, in many cases we can get useful results by considering the total excitation on the electron beam to consist of two space-charge waves: a fast, positive-power space-charge wave, and a slow, negative-power space-charge wave. This makes it possible to consider the electron beam as a transmission system supporting two waves.

We were able to prove some simple theorems about standing waves of zero power on such a transmission system. By assuming that the noise on the electron stream is due to a number of independent zero-power sources, we were able to find a simple expression for the noise standing wave on the electron stream.

Going beyond this, we were able to show how this formulation can be used to deduce certain results about noise in traveling-wave tubes.

Finally, there is the question of the effect of transverse velocities of electrons, both in special "transverse-field tubes" and in conventional traveling-wave tubes.

There is of course a great deal about noise in microwave tubes which has not been considered here. For one thing, there is the evaluation of the noise figure of many types of microwave tubes; here we have given only simple illustrative examples: the noise figure of a lossless traveling-wave tube with large space charge, and some theorems about noise cancellation.

Then, there is the effect of net noise power flow in the beam, which Haus has considered.

There is the effect of the actual physical configuration of the beam and the variation of noise across it which we disregarded by considering only two space-charge waves out of the many which can be excited. In a more realistic treatment of beam devices, transverse velocities of emitted electrons as well as longitudinal velocities should be included.

In other words, this paper constitutes a bare start in considering noise in microwave tubes.

APPENDIX

When electrons are accelerated from a cathode to form an electron beam they are acted on by certain average fields due to electrode voltages and space charge, and by certain fine-grained a-c fields due to the discrete nature of the charge, that is, to the fact that electrons are particles and not a smoothed-out jelly of charge.

The writer's judgment is that the velocity distribution in the electron flow is little affected by the fine-grained forces. If this is so, Liouville's theorem tells us that the electron density about any electron in a six-dimensional phase space is constant as the electron moves along its path.[37] If we assume a velocity distribution just outside of the body of the cathode which is like a Maxwellian distribution except that only electrons moving away from the cathode surface are present, the correct non-relativistic expression for the number density at any point in the electron flow is either zero or dn given by

$$dn = Ae^{-\left(\frac{mv^2}{2} - eV\right)/kt} dv_x dv_y dv_z \, dxdydz \qquad (A.1)$$

Here V is electric potential with respect to the cathode and

$$v^2 = v_x^2 + v_y^2 + v_z^2 \qquad (A.2)$$

From (A.1) we can calculate any moment of the velocity distribution provided that we know the boundary which separates regions in which the density is zero from regions in which the density is given by (A.1).

Two sorts of moments may be of interest. One refers to an average over the electrons in a unit volume. The other refers to electrons crossing a unit area in the x, y plane in a given time.

The first or volume average will be referred to by the subscript v for volume and the second or time average by the subscript t for time. The velocity distribution among electrons in a given volume is that usually dealt with in connection with gases. However, in the case of electron beams the distribution among electrons crossing a given plane in a given time is more important, for its moments are averages over the number of particles which flow through the tube and constitute the beam current. It is this average which is of interest in noise calculations.

One simple case is that of electric field in the z direction only. In this case the density is zero for

$$v_z^2 < 2\frac{e}{m} V \qquad (A.3)$$

and is given by (A.1) for larger values of v_z.

In this case we find the average, mean square, and mean-square deviations in velocity to be

$$\overline{v_{zv}} = \frac{\sqrt{2k/\pi m}\; e^{-eV/kt}}{1 - \operatorname{erf}\sqrt{eV/kT}} \tag{A.4}$$

$$\overline{v_{zt}} = \sqrt{2\frac{e}{m}V} + \sqrt{\frac{\pi kT}{2m}}\left(1 - \operatorname{erf}\sqrt{\frac{eV}{kT}}\right)e^{\frac{eV}{kT}} \tag{A.5}$$

$$\overline{v_{xv}} = \overline{v_{yv}} = \overline{v_{xt}} = \overline{v_{yt}} = 0 \tag{A.6}$$

$$\overline{v_{zv}^2} = \overline{v_{zv}}\sqrt{2\frac{e}{m}V} + \frac{kT}{m} \tag{A.7}$$

$$\overline{v_{zt}^2} = 2\frac{e}{m}V\left(1 + \frac{kT}{eV}\right) \tag{A.8}$$

$$\overline{v_{xv}^2} = \overline{v_{yv}^2} = \overline{v_{zt}^2} = \overline{v_{yt}^2} = \frac{kT}{m} \tag{A.9}$$

$$\overline{\delta v_{zv}^2} = \overline{v_{zv}^2} - (\overline{v_{zv}})^2 \tag{A.10}$$

$$\overline{\delta v_{zt}^2} = \overline{v_{zt}^2} - (\overline{v_{zt}})^2 \tag{A.11}$$

$$\overline{\delta v_{yv}^2} = \overline{v_{yv}^2} = \overline{\delta v_{yt}^2} = \overline{v_{yt}^2} \text{ etc.} \tag{A.12}$$

For very large values of V

$$\overline{v_{zv}} = \sqrt{2\frac{e}{m}V}\left(1 + (1/2)\left(\frac{kT}{eV}\right) - (3/4)\left(\frac{kT}{eV}\right)^2 + \ldots\right) \tag{A.13}$$

$$\overline{v_{zt}} = \sqrt{2\frac{e}{m}V}\left(1 + (1/2)\frac{kT}{eV} - (1/4)\left(\frac{kT}{eV}\right)^2 + \ldots.\right) \tag{A.14}$$

$$\overline{\delta v_{zv}^2} = (1/2)\left(\frac{kT}{m}\right)\left(\frac{kT}{eV}\right) \tag{A.15}$$

$$\overline{\delta v_{zt}^2} = (1/2)\left(\frac{kT}{m}\right)\left(\frac{kT}{eV}\right) \tag{A.16}$$

In many cases, the electrons from a large cathode are focused into an area or a beam of diameter small compared with the cathode. In such a case the mean square deviation in velocity may be considerably greater than in the case considered above, in which the fields act in the z-direction only.

If one assumes that the cathode is imaged perfectly on an area 1/M its linear dimension, it can be shown that the density in velocity space is zero inside an ellipsoid[21]

$$v_z^2 + (1-M^2)(v_x^2 + v_y^2) = 2\frac{e}{m}V$$

and for

$$v_z < 0$$

In this case, for large values of V, approximately

$$\overline{v_{xt}^2} = \overline{v_{yt}^2} = M^2\frac{kT}{m} \tag{A.17}$$

Thus, narrowing the beam increases the spread in transverse velocity. The mean square deviation in longitudinal velocity is again smaller than the spread in transverse velocity, but it is larger than in the case of a field in the z-direction only.

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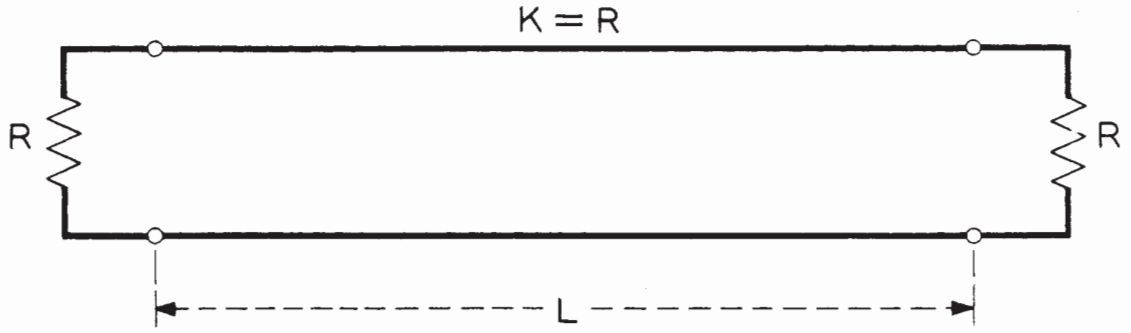

Fig. 1 - A lossless, non-dispersive transmission line of characteristic impedance K interconnects two resistors of resistance R which are held at a temperature T. A power kTB flows from each resistor to the other.

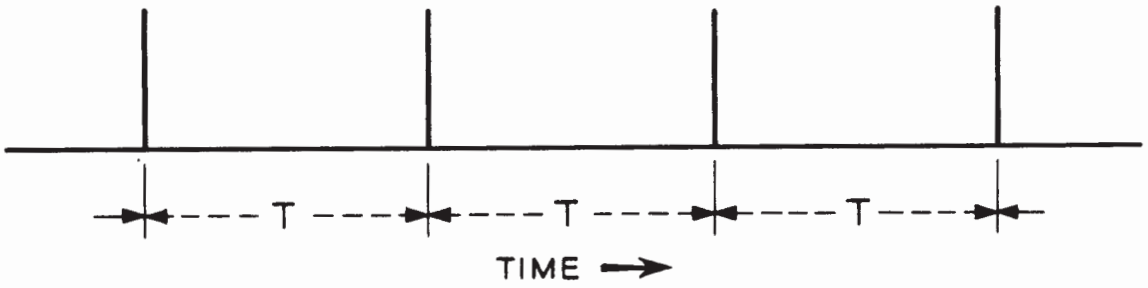

Fig. 2 - The passage of one electron each T seconds can be represented as a current of spiked wave form.

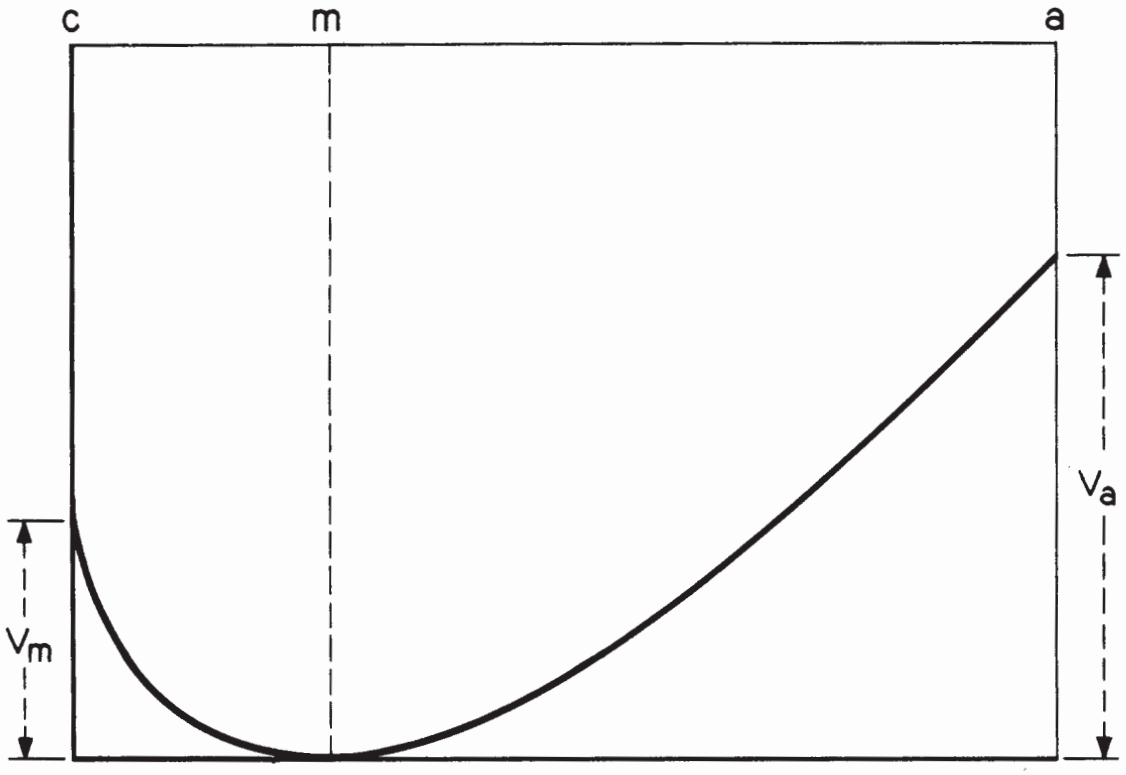

Fig. 3 - c, m and a indicate the positions of the cathode, the potential minimum and the anode. The curve indicates the variation of potential from cathode to anode. The cathode is V_m volts positive with respect to the potential minimum. The anode is V_a volts positive with respect to the potential minimum.

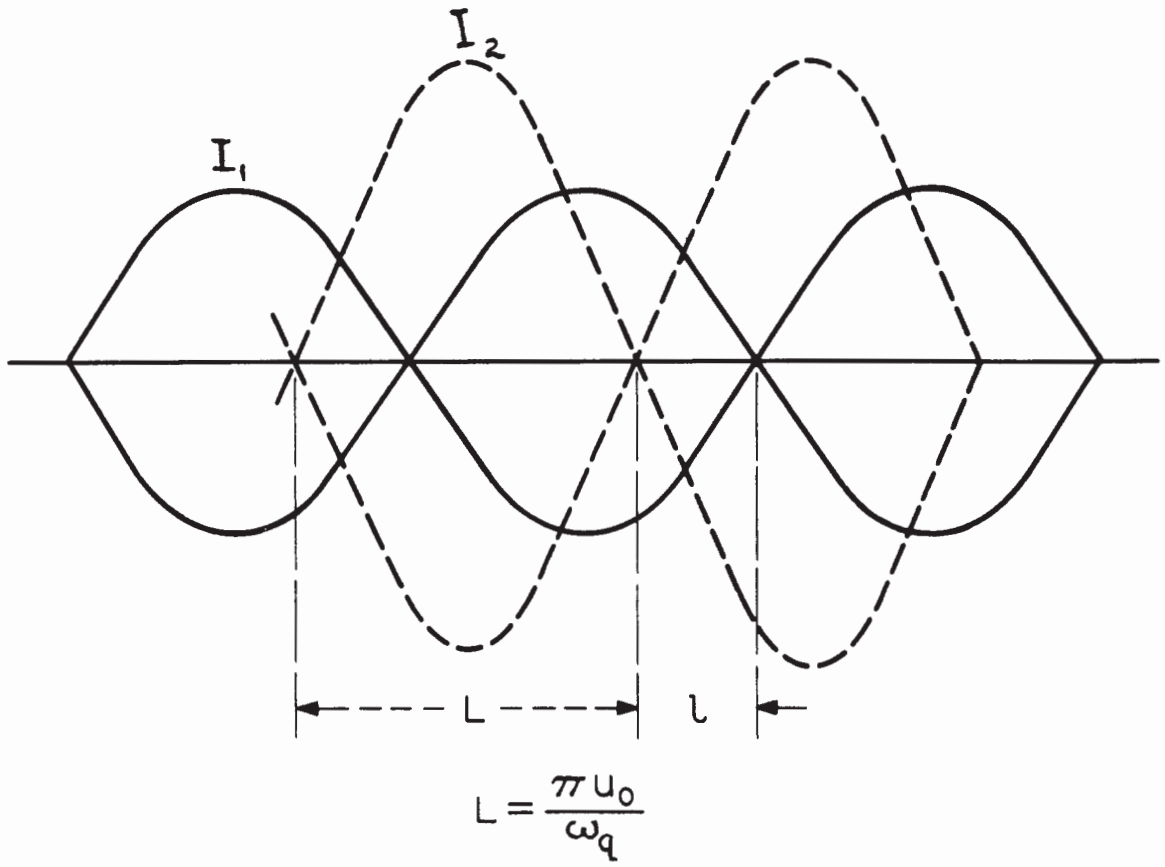

Fig. 4 - The envelopes of two uncorrelated zero power current standing-wave patterns, of peak currents maximum r.m.s. currents I_1 and I_2.